Technical Notes

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Subcooled Pool Film Boiling from a Cylinder and from a Sphere Placed in a Liquid Saturated Bed of Beads

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Nomenclature

A_c = surface area of cylinder for the experiments, Eq. (7)
A_s = surface area of cylinder or sphere over which integration is performed
d = bead diameter
D = cylinder or sphere diameter
I = current through the heater in the experiment
k = thermal conductivity
k_L = liquid thermal conductivity
$\bar{k}_L$ = effective thermal conductivity of porous bed in the liquid region
k_s = solid thermal conductivity
k_v = vapor thermal conductivity
$\bar{k}_v$ = effective thermal conductivity of porous bed in the vapor region
$\bar{k}_{v1}$ = effective thermal conductivity in vapor region for a single bead, Eqs. (5 and 6)
n = curvilinear coordinate, Fig. 1a
N = number of beads completely covered by vapor
Nu = Nusselt number, $\bar{q}D/[\bar{k}_v(T_w - T_s)]$
q = local heat flux at the wall
$\bar{q}$ = average heat flux at the wall, $\frac{1}{A_s}\int_{A_s} q \, dA_s$
Q = heat flux from the cylinder to the porous bed in the experiments
Q_{loss} = axial heat losses
r^* = quantity defined in Fig. 1b
R = cylinder or sphere radius
s = curvilinear coordinate, Fig. 1a
T = temperature
T_s = saturation temperature
T_w = wall temperature
T_∞ = liquid temperature far away from the body
V = voltage across the heater in the experiment
δ_v = vapor layer thickness
δ^* = thickness defined in Fig. 1b
ΔT_v = vapor superheat, $T_w - T_s$
ΔT_L = liquid subcooling, $T_s - T_\infty$
ϕ = porosity of porous matrix

Subscripts

L = liquid region
s = saturation or solid
v = vapor region
w = wall
∞ = liquid region unaffected by and far away from the vapor/liquid interface

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Introduction

SEVERAL engineering disciplines stand to benefit from a better understanding of boiling heat transfer in porous materials. These disciplines are exemplified by geothermal systems, solid matrix heat exchangers, the chemical and pharmaceutical industry, and the cooling of nuclear reactors and nuclear wastes in nuclear power applications.

It is an undisputed fact that the study of two-phase flows inside porous materials is complex both from the experimental and the theoretical viewpoints. In film boiling heat transfer, which is the subject of the present paper, perhaps the most important modeling assumption is that the two phases (liquid and vapor) are separated by a sharp and distinct boundary with no two-phase region in between.[1,2]

Boiling heat transfer in classical fluids has reached maturity and, to a certain degree, several fundamental questions in this area have been answered. The converse is true for boiling heat transfer in porous materials, which has attracted only limited attention. With reference to film boiling in particular, Cheng and Verma[2] studied theoretically the problem of pool film boiling from a flat plate in a porous medium. Orozco et al.[3] presented theoretical results for flow film boiling from a sphere or cylinder in a porous material. Experimental studies of boiling in porous media are scarce. Tsung et al.[4] carried out an investigation of boiling heat transfer from a sphere embedded in a porous medium composed of glass beads under steady-state and quenching conditions. It was found that the presence of the porous medium enhanced the overall heat

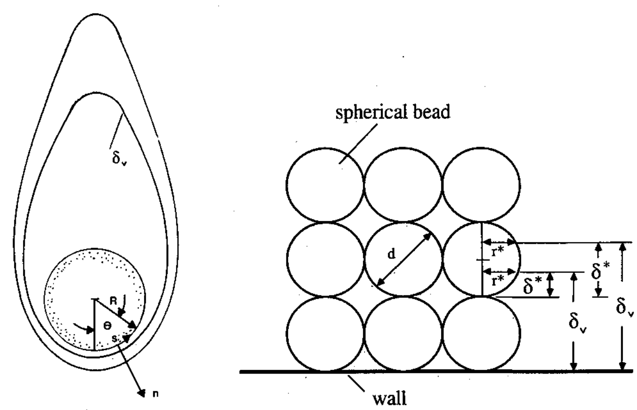

Fig. 1 Heat-transfer model and packing arrangement for the spherical beads.

transfer. Some of the results presented in Ref. 4 will be used in the present study to validate the theoretical model. Of particular relevance to the present study is the work of Cheng et al.[5] on film boiling about two-dimensional and axisymmetric bodies of arbitrary shape in porous medium. In this reference, the authors use the Darcy flow model and obtain results for the temperature field and heat-transfer rate from the heated surface. The effects of liquid subcooling and vapor superheat were also determined. The main goal of the present work is to extend the results in Ref. 5. This is achieved in the following manner: 1) Using a model for the porous medium effective thermal conductivity that accounts for the structure of the porous matrix (a bed of spherical glass beads) as proposed by Orozco et al.[6] 2) Comparing the new theoretical results to experimental findings for the case of pool boiling from a horizontal cylinder buried in a bed of glass beads. Theory and experiments proved to be in good agreement.

Theoretical Modeling and Solution Method

The schematic shown in Fig. 1a represents a cross section of a horizontal cylinder or a sphere of radius R under subcooled pool film boiling conditions. Choosing the curvilinear coordinate (s,n) makes the theoretical modeling for the two above-mentioned bodies identical. The body surface temperature is held constant at $T_w > T_s$. The liquid temperature far away from the vapor layer is also constant at $T_\infty < T_s$. Under the above conditions, the heat-transfer process from the cylinder or sphere to the liquid occurs by means of a double boundary-layer structure (Fig. 1a): the vapor layer adjacent to the body and the natural convection boundary layer surrounding the vapor layer. The two layers are separated by the phase-change interface.

All of the features of the theoretical modeling are included in detail in Cheng et al.[5] Therefore, they are omitted here for the sake of brevity. An important issue that needs to be addressed, however, is that pertaining to the effective thermal conductivity of the porous medium. A constant effective thermal conductivity was used in Cheng et al.[5] A more accurate representation of the effective thermal conductivity, in the vapor layer in particular, is likely to improve the theoretical modeling as well as the comparison between theoretical and experimental findings. The most popular model for the effective thermal conductivity in fluid-saturated porous media assumes that this conductivity is the volumetric average of the solid and the fluid. For example, if the fluid is vapor,

$$\bar{k}_v = \phi k_v + (1-\phi) k_s \tag{1}$$

Unfortunately, the above simple equation does not perform satisfactorily in the vapor region as shown by Orozco et al.[6] and Stellman.[7] The improved equation proposed by these authors for the average conductivity in the vapor-saturated region of a porous medium consisting of *spherical* beads, and written in terms of the quantities of the present study, is as follows:

If for the $(N+1)$th bead of the stack of beads N which are totally inside the vapor layer, $\delta^* \le d/2$ (Fig. 1b), then

$$\bar{k}_v = \delta_v \left(\frac{\delta^*}{\bar{k}_{v1}} + \frac{Nd}{A} \right)^{-1} \tag{2}$$

where

$$A = \left(1 - \frac{\pi}{4}\right) k_v + \frac{\pi}{4} \frac{2k_v k_s}{(k_v - k_s)^2} \left[k_v - k_s + k_s \ln\left(\frac{k_s}{k_v}\right) \right] \tag{3}$$

Similarly, if for the above-mentioned $(N+1)$th bead, $\delta^* > d/2$, then

$$\bar{k}_v = \delta_v \left(\frac{\delta^* - \dfrac{d}{2}}{\bar{k}_{v1}} + \frac{(2N+1)\dfrac{d}{2}}{A} \right)^{-1} \tag{4}$$

In Eqs. (2–4) $\bar{k}_{v1}$ is the average conductivity of the vapor layer for the case where the thickness of this layer is less than the diameter of a single bead, and it is obtained from the following relations:

For $\delta^* \le d/2$:

$$\bar{k}_{v1} = \frac{1}{d^2} \left\{ k_v (d^2 - \pi r^{*2}) + \frac{2k_v k_s}{(d-\delta^*)(k_v - k_s)^2} \left[(k_v - k_s)\delta^* + \left((k_s - k_v)\frac{d}{2} + k_v \delta^* \right) \ln\left(\frac{k_s}{k_v}\right) \right] \pi r^{*2} \right\} \tag{5}$$

For $\delta^* > d/2$:

$$\bar{k}_{v1} = \frac{1}{d^2} \left\{ k_v d^2 \left(1 - \frac{\pi}{4}\right) + k_s \pi r^{*2} + \frac{2k_v k_s}{\left(\delta^* - \dfrac{d}{2}\right)(k_v - k_s)^2} \left[(k_v - k_s)\delta^* + \left(\delta^* - \frac{d}{2}\right) k_s \ln\left(\frac{k_s}{k_v}\right) \right] \pi \left(\frac{d^2}{4} - r^{*2} \right) \right\} \tag{6}$$

The above equations for the effective thermal conductivity in the vapor layer were used in both the numerical calculations and the experimental measurements. For the liquid layer, the simple relation, Eq. (1), was sufficient to provide good agreement between theory and experiment.

After the governing partial differential equations were transformed into ordinary differential equations with the help of the similarity technique, they were solved numerically via a fourth-order Runge-Kutta shooting method as presented in Cheng et al.[5] However, in the present work we accounted for the dependence of the effective thermal conductivity on the thickness of the vapor layer [Eqs. (2–6)]. Because of space limitations, no details of the numerical procedure are presented here. They are included in Ref. 8.

Experimental Apparatus and Procedure

The heat-transfer measurements made in the present study were performed using an electrically heated cylinder placed inside a test section which consists of a rectangular stainless-steel box with glass windows on two sides so that visual observations of the phenomenon of interest could be made.

The cylinder consists of a stainless-steel rod of 12.7-mm outer diameter (o.d.). The cylindrical rod was machined to accommodate the installation of a 6.35-mm o.d., 54.8-mm long, high-power-density cartridge heater. Power to the heater was controlled by means of a Variac. The steady-state bulk temperature T_∞ of the porous medium was measured with three chromel-alumel thermocouples, located directly above, to the side, and directly below the test specimen. Thermocouples were also attached at selected locations on the heated surface and at a distance of 1.59 mm below the surface. The steady-state surface temperature T_w, corresponding to an arbitrarily imposed overall rate of heat dissipation Q, was measured by averaging the readings of the surface temperatures. It was observed that during partial film boiling the heated surface would experience temperature variations as large as 50°C between the forward and the backward stagnation points. During nucleate boiling, negligible surface-temperature gradients were observed. The axial heat losses were computed by means of the temperature differences measured at the ends of the heated area. All thermocouple outputs were recorded by a Hewlett Packard computer-controlled data acquisition system.

For a given power input to the heater, the heat flux Q from the cylinder to the fluid-saturated porous bed was calculated from

$$Q = (VI - Q_{\text{loss}})/A_c \tag{7}$$

The heat losses were never more than 5% of the total power input.

The porous medium consisted of 3-mm glass beads. All properties of this medium were measured: the porosity $\phi = 0.39$, the effective (liquid-saturated) thermal conductivity $k = 0.97$ W/m°C, and the permeability $K = 2\times10^{-9}\text{m}^{-2}$. Freon 113 was used as the working fluid.

With reference to the spherical geometry, the data published by Tsung et al.[4] were used for comparison with the theoretical model.

Results and Discussion

All of the fluid properties used in the theoretical calculations were for Freon 113. This facilitated the comparisons between theory and experiments at the end of the study. The Nu dependence on subcooling and superheat is illustrated in Fig. 2. For the same amount of subcooling and superheat, the Nu values for the cylinder are consistently lower than those for the sphere. Increasing the subcooling increases the value of Nu for both bodies of interest. Increasing the superheat, on the other hand, seriously decreases the value of Nu. This effect is more severe for the smaller values of superheat in Fig. 2 ($100°\text{C} < \Delta T_v < 200°\text{C}$). An important clarification is needed at this point. The decrease in Nu with ΔT_v does not mean that the average heat flux from the cylinder or sphere decreases with ΔT_v. Our results for $\bar{q}$, not shown here for brevity, indicated that quite the opposite is true. However, based on the definition of Nu this fact proves that the dependence of $\bar{q}$ on $\Delta T_v(T_w - T_s)$ is weaker than linear, thus explaining the decrease of Nu with increasing ΔT_v.

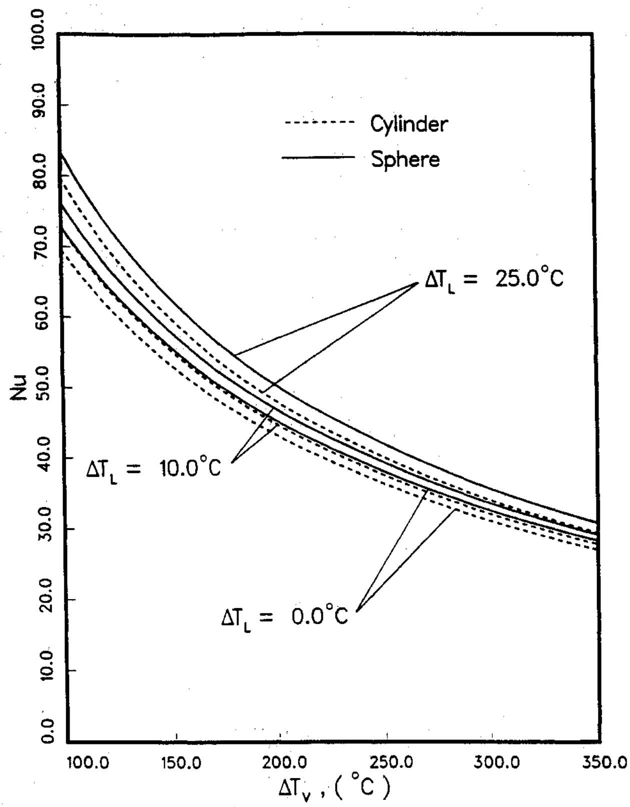

Fig. 2 Nusselt-number dependence on vapor superheat and liquid subcooling ($d/D = 2.36\times10^{-3}$).

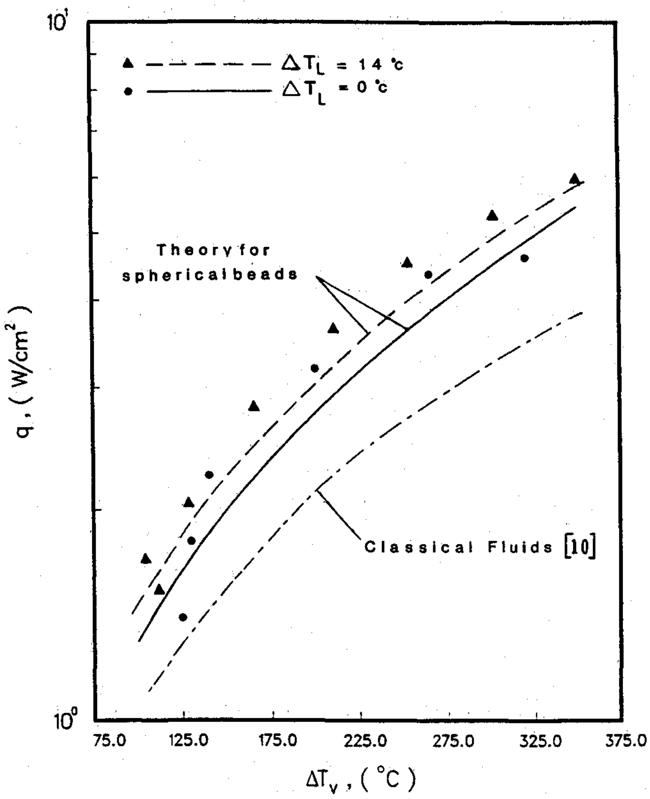

Fig. 3a Comparison of average heat-transfer flux between the theoretical model for a cylinder placed in a liquid-saturated bed of beads, a cylinder in classical saturated pool film boiling, and experimental data ($d/D = 2.36\times10^{-1}$).

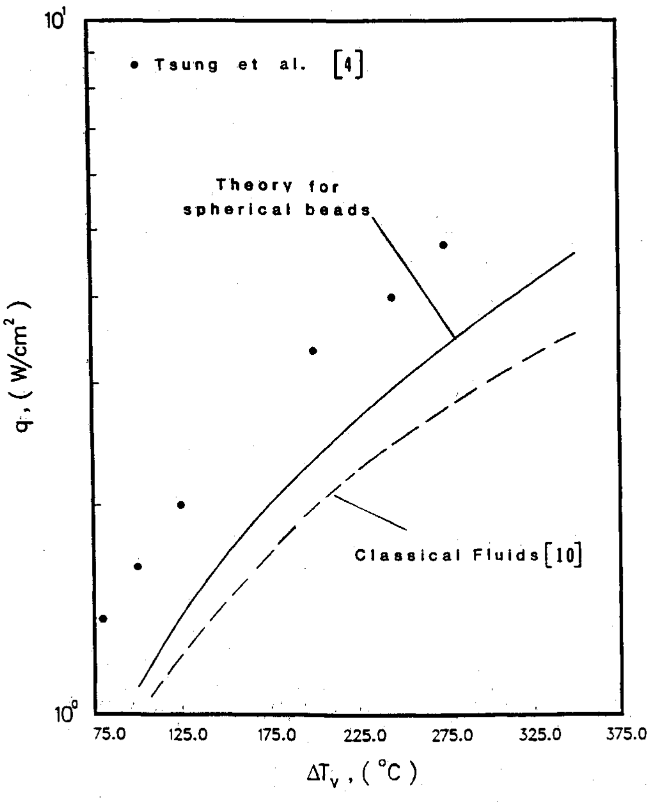

Fig. 3b Comparison of average heat-transfer flux between the theoretical model for a sphere embedded in a liquid-saturated bed of beads, a sphere in classical saturated pool film boiling, and the experimental data of Tsung et al.[4] for zero subcooling ($d/D = 2.36\times10^{-1}$).

A comparison between theory and experiment is reported in Fig. 3a for the cylinder. Considering the approximate nature of the theoretical model, the agreement is good for both cases of subcooling and no subcooling. The theory underpredicts the experimental findings somewhat, rather consistently. Two reasons for this fact are speculated. First, the point-contact assumption between the glass beads and the cylinder surface was likely violated somewhat in the experiments, thus increasing the thermal conductivity and yielding higher average heat transfer. Second, probable collapses in the vapor film (partial film boiling) in the back of the cylinder contributed to increasing the average heat flux in the experiments.

Also shown in Fig. 3a is a theoretical prediction for the average heat flux for pool film boiling without subcooling in classical fluids.[7] This average heat flux is lower than what was found in the present study for a cylinder embedded in a bed of glass beads. This result indicates that porous media exhibit a clear potential as heat-transfer augmentation devices if they are made out of a rather conductive material (the glass beads are considerably more conductive than the vapor) and are porous enough to allow for fluid motion inside the solid matrix (the porosity of a bed of spheres, approximately 40%, is reasonable).

Figure 3b shows a comparison between the present theoretical modeling for the sphere and experimental results for the case of zero subcooling reproduced from Ref. 4. No results for subcooled film boiling were available in Ref. 4. An approximate theoretical solution for the average heat flux in classical fluids[7] is also reported. The main conclusion to be drawn from Fig. 3b for the spherical geometry is similar to that of Fig. 3b for the cylinder. For brevity, the discussion is not repeated.

Overall, the experiments exhibited good agreement with the present theoretical predictions for both geometries of interest. Comparison with results for classical fluids indicated that a potential exists in the use of conductive porous materials in heat-augmentation devices.

Acknowledgments

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References

[1]Cheng, P., "Film Condensation Along an Inclined Surface in Porous Medium," *International Journal of Heat and Mass Transfer*, Vol. 24, No. 6, 1981, pp. 983–990.

[2]Cheng, P. and Verma, A. K., "The Effect of Subcooled Liquid on Film Boiling Above a Vertical Heated Surface in a Porous Medium," *International Journal of Heat and Mass Transfer*, Vol. 24, No. 7, 1981, pp. 1151–1160.

[3]Orozco, J., Poulikakos, D., and Gutjahr, M., "Flow Film Boiling from a Sphere and from a Horizontal Cylinder Embedded in a Porous Medium," *Journal of Thermophysics and Heat Transfer*, Vol. 2, No. 4, 1988, pp. 359–364.

[4]Tsung, V. X., Dhir, V. K., and Singh, S., "Experimental Study of Boiling Heat Transfer from a Sphere Embedded in Liquid Saturated Porous Media," *Heat Transfer in Porous Media and Particulate Flows*, HTD-Vol. 46, edited by L. S. Hao, 1985, American Society of Mechanical Engineering, New York, pp. 127–134.

[5]Cheng, P., Chiu, D. K., and Kowk, L. P., "Film Boiling About Two-Dimensional and Axisymmetric Isothermal Bodies of Arbitrary Shape in a Porous Medium," *International Journal of Heat and Mass Transfer*, Vol. 25, No. 8, 1982, pp. 1247–1249.

[6]Orozco, J., Stellman, R., and Gutjahr, M., "Film Boiling Heat Transfer from a Sphere and a Horizontal Cylinder Embedded in a Liquid Saturated Porous Medium," *Journal of Heat Transfer*, Vol. 110, No. 4(A), 1988, pp. 961–967.

[7]Stellman, R., "Study of Film Boiling Heat Transfer from a Cylinder and a Sphere Including Porous Media," M.S. Thesis, Univ. of Illinois at Chicago, IL, 1987.

[8]Gutjahr, M., "Analysis of Natural Convection Film Boiling," M.S. Project, Univ. of Illinois at Chicago, Chicago, IL, 1987.

Emittance of a Finite Spherical Scattering Medium with Fresnel Boundary

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Introduction

THE analysis of radiative transfer within a medium with a reflecting boundary has been a subject of great interest.[1] Recently, several studies were performed to examine the influence of specular and diffuse reflection on radiative transfer with various geometries. Pomraning and Siewert[2] reported the integral form of the equation of radiative transfer for a sphere with a specularly and diffusely reflecting boundary; Thynell and Özisik[3] developed exact integral expressions for radiative transfer in a cylindrical medium with a diffusely and specularly reflecting boundary. Pomraning[4] estimated the emittance from a half-space with a specularly and diffusely reflecting boundary, whereas Lin and Huang[5] studied the directional emittance for an infinite cylinder by the Galerkin method.

Since the radiative heat transfer in semitransparent spherical bodies occurs in many industrial processes such as droplet vaporization and combustion, glass making, and growing of artificial crystals, radiative transfer in a sphere with a Fresnel boundary is of practical importance. This study is intended to show the effects of scattering albedo, refractive index, and optical radius as well as the effects of geometry on the directional emittance for a spherical medium. The ray-tracing technique[2] is adopted to develop the exact integral expressions for radiative transfer in an isotropically scattering sphere with a Fresnel boundary, in which the reflectivity is directionally dependent. The integral form of the equation of radiative transfer is solved by approximating the integral term by a Gaussian quadrature. Exact expressions are then used for the prediction of the directional emittance.

Analysis

The system considered is an absorbing, emitting, and isotropically scattering sphere of a finite optical radius R, which is much greater than radiation wavelength. Assume that the medium is isotropic, homogeneous, isothermal, and in local thermodynamic equilibrium, and that Fresnel reflection is included at the boundary. The equation of radiative transfer and the boundary condition are

$$\mu\frac{\partial I(r,\mu)}{\partial r}+\frac{1}{r}(1-\mu^2)\frac{\partial I(r,\mu)}{\partial \mu}$$
$$+I(r,\mu)=(1-\omega)n^2 I_b(T)$$
$$+\frac{\omega}{2}\int_{-1}^{1} I(r,\mu')\,\mathrm{d}\mu'$$
$$\text{in}\quad 0<r<R,\qquad -1\le\mu\le 1 \tag{1}$$

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